THE MOTION OF A PARTICLE IN A VISCOUS MEDIUM DISCHARGING AS A JET

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Inzhenerno-Fizicheskii Zhurnal, Vol. 11, No. 5, pp. 569–574, 1966

UDC 532.58

Motion of a particle is examined during jet discharge of a viscous liquid in the region Re ≤ 300 with a hydrodynamic resistance coefficient which takes account of all the inertial terms.

To solve practical problems connected with jet discharge of dust-laden gases and liquids, in which there are solid particles, we need to determine the mean free path of a particle, its relative velocity, as well as the time during which the particle or the drop of liquid has this relative velocity.

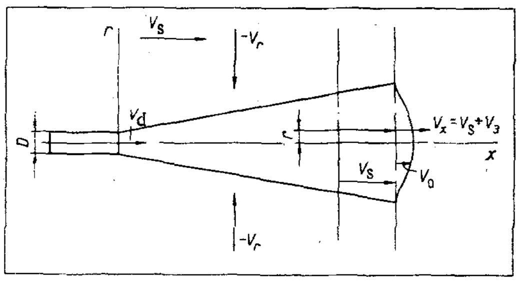

Fig. 1. Schematic of the motion in a circular free jet.

Exact calculation of the trajectory of a particle in the aerodynamic resistance force field of a jet is quite a difficult problem. An attempt was made in reference in [1] to determine the trajectory of a particle in such a field. However, in the calculations a hydrodynamic resistance coefficient ψ was assumed for the medium, partially taking into account the inertia terms. It was shown in [2, 3] that use of a coefficient ψ, partially taking account of inertia terms (V_0 = const) for the decelerated motion of the particles ($V_0 \to 0$) leads to considerable errors, which increase with increase of the Reynolds number (Re → 300).

We shall examine the motion of a particle in a circular free jet, issuing into a secondary stream, which moves with constant velocity (V_s = const), in the region Re ≤ 300. We shall assume that the flow possesses axial symmetry, and the x axis coincides with the axis of the jet. The coordinate origin is located in the center of the plane of the aperture from which the jet issues (Fig. 1).

The external pressure stream at the jet boundary is constant and therefore the pressure gradient $\partial p_0/\partial x = 0$ (since V_s = const). The velocity of the stream V(x, r) in the circular free jet may be calculated by one of the theories described in references [4–10].

For the case under examination we accept the diffusion method of vorticity transport described in [4, 10], which gives satisfactory agreement with experimental data. The essence of this method is that the differential equations of motion for the momentum $\rho_1 V_2^2$ must be analogous to the equations of molecular diffusion or heat conduction, since experimental measurements have shown that the distribution of velocities in transverse sections along the path of the mixing (the x axis) are a good approximation to the Gaussian error curve. Therefore, to solve the differential equations the following condition is assumed: the intensity of transport of momentum corresponding to component V_r in the transverse direction (along the r axis), with velocity V_x is proportional to the change of the momentum flux $\rho_1 V_r^2$ in this transverse direction, i.e.,

$$\overline{V}_x \overline{V}_r = -L(x) \frac{\partial \overline{V}_x^2}{\partial r}. \tag{1}$$

The equation of motion in the projection on the x axis is as follows:

$$\rho \left(\overline{V}_x \frac{\partial \overline{V}_x}{\partial x} + \overline{V}_r \frac{\partial \overline{V}_x}{\partial r} \right) = \frac{1}{r} \frac{\partial}{\partial r} (r, \sigma_{x,r}). \tag{2}$$

Substituting the values $\overline{V}_x = \overline{V}_s + \overline{V}_2$ into (2) we obtain

$$\left(\frac{V_d}{V_d} + \frac{\overline{V}_2}{V_d} \right) \frac{\partial}{\partial x} \frac{\overline{V}_2}{V_d} + \frac{\overline{V}_2}{V_d} \frac{\partial}{\partial r} \frac{\overline{V}_2}{V_d} = 0.5 \frac{\partial}{\partial r} \left(r, \frac{\sigma_{x,r}}{\rho_1, V_d^2} \right). \tag{3}$$

Neglecting terms containing the viscosity and the pressure, and also taking ρ_1 = const, we shall write the equation of conservation of momentum on the basis of (3) to be

$$\frac{\partial}{\partial x} V_x^2 = \frac{1}{r} \frac{\partial}{\partial r} (r, \overline{V}_x, V_2). \tag{4}$$

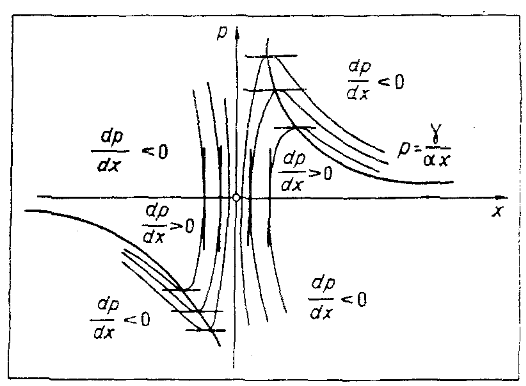

Fig. 2. Qualitative picture of the behavior of particle trajectories in the phase plane pox.

Assuming $V_x^2 = V_s^2 + V^2$, where $\overline{V}^2 = 2V_s\overline{V}_0 + \overline{V}_0^2 + \overline{U}^2$, Eq. (4) may be transformed into the following form:

$$\frac{\partial}{\partial x}\overline{V}^2 = L(x)\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \overline{V}^2}{\partial r}\right). \tag{5}$$

In the case when the distribution of velocities is similar in successive cross sections, we may write

$$\frac{\overline{V}_2}{\overline{V}_{2,\max}} = f(\zeta_2), \quad \frac{\overline{V}_r}{V_{2,\max}} = f_1(\zeta_2), \quad \frac{\overline{V}_{2,\max}}{V_d} = f(\zeta_1). \tag{6}$$

This condition of similarity of profiles is observed in two cases: when $\overline{V}_2/V_s \gg 1$, and also when $\overline{V}_2/V_s \ll 1$. We shall examine the case when $V_s = 0$.

Assuming $\overline{V}^2/\overline{V}_d^2 = f(\zeta_1)f(\zeta_2)$, where ζ_1 and ζ_2 are determined by relations (6), we obtain a solution of Eq. (5)

$$\overline{V}(x, r) = CV_d\frac{D}{x+a}\exp\left(-\frac{x+a}{2L(x)}\zeta_2^2\right)^{0.5}. \tag{7}$$

From Eq. (7) we may obtain the value of the velocity $V(x, 0)$ on the jet axis ($\zeta_2 = 0$)

$$\overline{V}(x, 0) = CV_d\frac{D}{x+a}. \tag{8}$$

Having determined the law of variation of velocity $\overline{V}(x, r)$ along the x axis, we shall write down the differential equation of motion of a particle in the field of the aerodynamic resistance force of the medium for the region Re < 1 ($\psi = 24/\mathrm{Re}$):

$$m\frac{dV_r}{dt} = 3\pi\eta\, dV_0, \tag{9}$$

where V_0 is the relative velocity of the stream with respect to the particle

$$\overline{V}_0 = \overline{V}(x, r) - \overline{V}_2. \tag{10}$$

Substituting the value $\overline{V}_0$ from (10) into (9) and assuming $\overline{V}(x, r) = \overline{V}(x, 0)$, we may write, following appropriate transformations, the differential equation of motion of the particle along the jet axis in the following form

$$x(x'' + \alpha x') = \gamma, \tag{11}$$

where $\alpha = 18\eta/d^2\rho_2$; $\gamma = CV_d/Dd$.

The solution of the non-linear equation (11) may be written as

$$x = \frac{\gamma\exp(z^2/2\gamma)}{\alpha\int\exp(z^2/2\gamma)\,dz}, \tag{12}$$

where $z = \alpha x + x'$.

Equation (12) cannot be solved in elementary functions and may be solved graphically. However, without loss of generality and without large errors, an approximate solution of (11) may be found.

We shall analyze the behavior of the trajectories (integral curves) of motion of a particle described by the differential equation (11). To do this we shall construct a picture of phase trajectory in the phase plane.

By the substitution $p = x'$, $pp' = x''$ we shall transform Eq. (11) to the following form:

$$\frac{dp}{dx} = \frac{\gamma}{px} - \alpha. \tag{13}$$

Analysis shows that $dp/dx = 0$ determines a hyperbola. When $dx/dp = 0$, $-dp/dx = \infty$ we have vertical tangents. There are no singular points. The axis $p = \gamma/\alpha x$, which in turn approaches the x axis asymptotically. This indicates that the particle velocity V_0 asymptotically approaches zero with increasing distance from the nozzle.

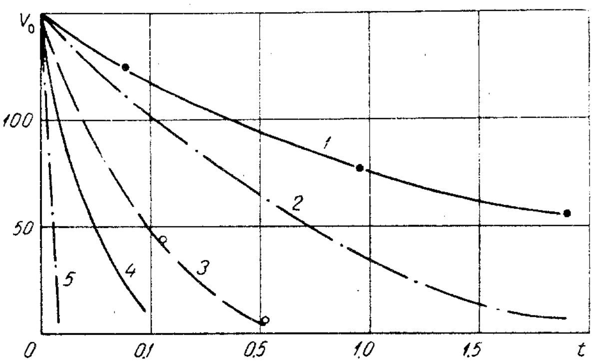

Fig. 3. Relative particle velocity V_0 (m/sec) versus the time t (sec) for d (μ): 1—500; 2—200; 3—100; 4—50; 5—10.

For an approximate solution of Eq. (11) we shall use a method of successive integration, taking the gas velocity at each time step to be constant and equal to its value at the beginning of the step. Then, by differentiating (10) with respect to time, and substituting the value of dV_2/dt into Eq. (11), following the appropriate transformations, we obtain

$$\frac{dV_0}{V_0} = -\alpha\, dt. \tag{14}$$

Integrating (14) within the limits $t_1 = 0$ and $t_2 = t$ and the velocity from V_0 to $V_0 = V(x, r)$, we obtain

$$V_0 = V(x, r)\exp(-\alpha t). \tag{15}$$

As is seen from Fig. 3, particles with $d \le 50\ \mu$ are decelerated by the medium in the course of 0.12 sec (the value of relative velocity between the particle and the stream is approximately zero after this time interval). Particles with $d \le 10\ \mu$ are decelerated by the medium in a time ~0.01 sec.

The motion of coarser particles ($d > 50\ \mu$), however, takes place in the region Re ≤ 300. It was shown in references [2, 3] that the use of a hydrodynamic resistance coefficient of the medium, partially taking into account inertia terms (V_0 = const) during the deceleration of the particles, and also the use of the coefficient $\psi = 24/\mathrm{Re}$ for the region Re ≫ 1 leads to considerable errors.

During jet discharge of dust-laden gases, as the result of lowering of the velocity of the gas, a relative velocity arises between the particle and the stream. The value of the relative velocity V_0 varies from the highest to the lowest values ($V_0 \to 0$), since the particle is decelerated by the medium.

The differential equation of motion of a particle in the region Re ≤ 300 is as follows:

$$m\frac{dV_2}{dt} = \left(A + \frac{B}{\mathrm{Re}}\right)\frac{\pi d^2}{8}\rho_1 V_0^2, \tag{16}$$

where A and B are constants equal to 0.12 and 37, respectively, for the region $Re \leq 300$ [2], taking into account inertia terms during the decelerated change of relative velocity. For more exact calculation we must take A and B equal to 0.055 and 50 for the region $100 < Re \leq 300$; 0.805 and 37 for the region $6 < Re \leq \leq 100$; and 4.45 and 24 for the region $Re \leq 6$ [2].

Taking $V(x, r) = \text{const}$ for successive integration (for a chosen pitch), we shall transform the differential equation (16) to the following form:

$$dt = -\frac{dV_0}{\beta V_0^2 + \varphi V_0}, \tag{17}$$

where $\beta = 0.75 A \rho_1 (d \rho_2)^{-1}$; $\varphi = 0.75 B \eta (d^2 \rho_2)^{-1}$.

Integrating (17) within the same limits as in (14), we obtain

$$t = \frac{1}{\varphi} \ln \left| \frac{V_0 [1 + \zeta V(x, r)]}{V(x, r) [1 + \zeta V_0]} \right|, \tag{18}$$

where $\zeta = Ad(B\nu)^{-1}$.

Knowing the value of velocity V(x, r) (Eqs. (7) and (8)), we choose a value of pitch, depending on the required degree of accuracy, and determine the particle trajectory by a successive computation method. Moreover, the equations (15) and (18) obtained allow us to determine the relative velocity and the time during which the particle has the relative velocity.

By way of example we shall determine the time t for which the relative velocity attains the values $V_0 = = 50$ m/sec for a particle with $d = 100\ \mu$ for $V(x, 0) = = 150$ m/sec, $\rho_2 = 2000$ kg/m^3, $\eta = 1.98 \cdot 10^{-5}$ N $\cdot$ sec/m^2, $\rho_1 = 1.3$ kg/m^3. From Eq. (18), $t = 0.813$ sec. For the same conditions, but from Eq. (15), $t = = 0.061$ sec.

It is seen from the example given that considerable errors arise in using Eq. (15) to calculate the time t when the particle in fact is moving in the region $Re > > 1$.

NOTATION

V_x, V_r—components of absolute velocity of the jet in a cylindrical coordinate system x, r, m/sec; V_s—absolute velocity of a secondary stream into which the jet discharges, m/sec; V_3—excess velocity V_s, i.e., $V_x = V_n + V_0$, m/sec; V_d—discharge velocity of the jet from the nozzle, m/sec; V_2—particle velocity, m/sec; V_0—relative velocity of the particle, m/sec; η, ν—viscosity of the medium, N·sec/m^2 and m^2/sec; ρ_1, ρ_2—densities of the medium and of the particle, kg/m^3; d—particle diameter, m; t—time, sec; D—nozzle diameter, m; $\zeta_1 = (x + a)/D$; $\zeta_2 = r/[D(x + a)]^{0.5}$; a, C—constants.

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5 April 1966

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